Modeling Sprint Cycling Using Field-Derived Parameters and Forward Integration

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ABSTRACT

MARTIN, J. C., A. S. GARDNER, M. BARRAS, and D. T. MARTIN. Modeling Sprint Cycling Using Field-Derived Parameters and Forward Integration. Med. Sci. Sports Exerc., Vol. 38, No. 3, pp. 592–597, 2006. We previously reported that a mathematical model could accurately predict steady-state road-cycling power when all the model parameters were known. Application of that model to competitive cycling has been limited by the need to obtain accurate parameter values, the non–steady-state nature of many cycling events, and because the validity of the model at maximal power has not been established. Purpose: We determined whether modeling parameters could be accurately determined during field trials and whether the model could accurately predict cycling speed during maximal acceleration using forward integration. Methods: First, we quantified aerodynamic drag area of six cyclists using both wind tunnel and field trials allowing for these two techniques to be compared. Next, we determined the aerodynamic drag area of three world-class sprint cyclists using the field-test protocol. Track cyclists also performed maximal standing-start time trials, during which we recorded power and speed. Finally, we used forward integration to predict cycling speed from power–time data recorded during the maximal trials allowing us to compare predicted speed with measured speed. Results: Field-based values of aerodynamic drag area (0.258 ± 0.006 m²) did not differ (P = 0.53) from those measured in a wind tunnel (0.261 ± 0.006 m²). Forward integration modeling accurately predicted cycling speed (y = x, r² = 0.989) over the duration of the standing-start sprints. Conclusions: Field-derived values for aerodynamic drag area can be equivalent to values derived from wind tunnel testing, and these values can be used to accurately predict speed even during maximal-power acceleration by world-class sprint cyclists. This model could be useful for assessing aerodynamic issues and for predicting how subtle changes in riding position, mass, or power output will influence cycling speed. Key Words: WORLD CLASS, AERODYNAMICS, WIND TUNNEL, MAXIMAL POWER

Mathematical cycling models (4,5,8,12,14) allow scientists, coaches, and athletes to systematically examine the extent to which alterations to various aspects of the cyclist, his or her bicycle, or environmental conditions might alter cycling performance. Although individual models include specific nuances, most include terms for power produced by the cyclist and power required to overcome aerodynamic drag, rolling resistance, drive train friction, and to accelerate or raise the cyclist’s center of mass. The relative importance of each of those terms depends on the instantaneous conditions, but for steady-state riding over relatively flat terrain, aerodynamic drag has been reported to be the dominant term, requiring up to 96% of the cyclist’s power (12). Consequently, minimizing aerodynamic drag is paramount in efforts to optimize cycling performance. For other conditions, such as maximal acceleration from a standing start or cycling up steep grades, changes in kinetic or potential energy will consume most of the cyclist’s power (8). Furthermore, a cyclist can experience several conditions within a single competitive event. During a 1-km track cycling time trial, for example, power produced at the start will mostly act to accelerate the rider, whereas power produced later in the race will mostly act to overcome aerodynamic drag. Thus, decisions regarding the relative importance of aerodynamic drag and body or bicycle mass, and the timing of application of power (i.e., pacing strategies) require careful analysis that can be facilitated with mathematical modeling.

Previously, we (12) reported that a mathematical model could account for 97% of the variability in steady-state road cycling power when all the model parameters (aerodynamic drag, rolling resistance, friction in the bearings and chain drive system, and changes in kinetic and potential energy) were known. Measuring these parameters, however, requires sophisticated wind tunnel (12), tire (9), and bearing testing (3). Additionally, although our model predicted cycling power quite well, our measurement intervals were fairly large (472 m) and our model did not account for rapid changes in power or speed. Finally, as others have done (13,14), we focused on modeling endurance cycling (~200 W) rather than sprint cycling where maximal power can exceed 1000 W. It is worth pointing out that of the eight
world championship time-trial events held by the Union Cycliste Internationale (six individual and two team events), only two (men’s and women’s road individual time trials) are performed at submaximal steady-state intensities. The remaining six events (500-m time trial, 1000-m time trial, 3000-m pursuit, 4000-m pursuit, team pursuit, and team sprint) are performed at intensities beyond maximal aerobic power output, with three of these being truly maximal (2). Thus, whereas our previous model has been useful (8,10,11), its application has been limited by the need to obtain accurate parameter values, the non–steady-state nature of many cycling events, and because the validity of the model at maximal power has not been established.

In this investigation, our goal was to broaden the application of our previous model by adapting it to field-based parameters and by applying it to non–steady-state, maximal-power cycling. More specifically, we designed a series of experiments to determine whether model parameters could be accurately quantified during field tests, and whether a model, using field-based parameters, could accurately predict cycling speed during high-power, non–steady-state cycling. We hypothesized that values for aerodynamic drag could be accurately derived from field test data and that speed during non–steady-state cycling could be accurately modeled using a simple linear forward integration technique.

METHODS

Overview. To accomplish our stated objectives we performed three discrete tasks. First, we modified our previous model so that it depended only on two coefficients: aerodynamic drag area (coefficient of drag \(C_{DA}\)) and a global coefficient of friction (\(\mu\)). Second, we determined those coefficients from field test data and compared them with \(C_{DA}\) values measured in a wind tunnel and with previously reported values for coefficients of rolling resistance (\(C_{RR}\)). Third, we used field-derived coefficients and forward integration to predict cycling speed from known power–time data and compared predicted speed with measured speed to establish the validity of the forward integration technique. For these final trials, we recorded data during maximal standing-start accelerations performed by world-class sprint cyclists.

Task 1: Model modification. In our previous model (12), we expressed power as a function of aerodynamic drag, rolling resistance, bearing friction, changes in kinetic and potential energy, and drive system efficiency. Each term in the model was a function of either ground speed \(S_g\) or the product of air speed squared and ground speed \(S_a S_g^2\). Consequently, we rewrote our original equation to include only two coefficients:

\[
\text{Power} = [C_{DA} \times \left(\frac{\rho S^2_a}{S_g}\right) + \mu \times (S_a F_N)] + \Delta PE/\Delta t + \Delta KE/\Delta t \times E
\]

in our previous model, \(E\) is efficiency of the drive system (assumed to be 97.7% based on our previous findings (12)), \(PE\) is potential energy, \(KE\) is kinetic energy, \(\rho\) is air density, and \(F_N\) is the normal force exerted by the bicycle tires on the rolling surface (essentially weight of the bicycle and rider). Equation 1 was rearranged to form an expression with only the two resistance terms on one side and the power and energy terms on the other:

\[
P \times E - \Delta PE/\Delta t - \Delta KE/\Delta t = C_{DA} \times \left(\frac{\rho S^2_a}{S_g}\right) + \mu \times (S_a F_N)
\]

Using this form of the equation, we can determine \(C_{DA}\) and \(\mu\) via regression analysis.

Task 2: Parameter determination and comparison. In our previous study (12), we obtained \(C_{DA}\) of six male cyclists from wind tunnel testing conducted at an air speed of approximately 13.4 m s\(^{-1}\) and at yaw angles (i.e., the angle of alignment between the bicycle and the air stream) of 0, 5, 10, and 15\(^\circ\). In that previous study, each cyclist also performed cycling trials on a straight concrete surface with a grade of 0.3%, during which we measured bicycle ground speed, cycling power, air speed, and wind direction. Each cyclist rode the test section in both directions at three different speeds (approximately 7, 9, and 11 m s\(^{-1}\)), and one subject rode at a fourth speed (12 m s\(^{-1}\)).

In the present study, we reused those previously obtained data to produce field-derived values for \(C_{DA}\) and \(\mu\). Specifically, we used those power, ground speed, wind speed, air density, and energy data, and the assumed value for drive system efficiency, to determine field-based \(C_{DA}\) and \(\mu\) values in a two-step process. First, we determined a value for the quantities on the left side of Equation 2 \((P \times E - \Delta PE/\Delta t - \Delta KE/\Delta t)\) and for several terms on the right side of the equation \((1/2 \rho S^2_a S_g + S_a F_N)\) for each trial, and determined \(C_{DA}\) and \(\mu\) for each subject via multiple linear regression. We then entered the mean value for \(\mu\) into Equation 2 and determined individual \(C_{DA}\) values via linear regression. We used this two-step process to establish a single value for \(\mu\), which we expected to be constant for a specific bicycle tire and surface combination.

Task 3: Forward integration modeling of non–steady-state cycling performance. Three world-class sprint cyclists (subject 1, a male match-sprint specialist: 1.83 m, 96 kg; subject 2, a male kilometer time-trial specialist: 1.82 m, 87 kg; and subject 3, a female 500-m specialist: 1.65 m, 68 kg) volunteered to participate in this phase of the investigation. Each subject had been world champion at least once in a track sprint cycling event, and all were Olympic medalists. We explained the requirements of the investigation to each cyclist, and she or he gave written informed consent. The Australian Institute of Sport human research ethics committee approved the methods used in this investigation. These three cyclists only performed field trials; they did not participate in wind tunnel testing. Each cyclist performed steady-state trials (6–21 s) on an indoor velodrome at speeds ranging from 6 to 16 m s\(^{-1}\) in seated and standing positions while power and speed were recorded.
at a frequency of 5 Hz with a dynamically calibrated professional version Schoberer Resistance Mechanism (SRM) Powermeter (Germany). We determined the modeling coefficients $C_D A$ and $\mu$ from the steady-state data, as described above. The steady-state trials used to calculate $C_D A$ and $\mu$ were performed on a 250-m velodrome (Superdrome, Adelaide, South Australia) that is approximately oval in shape. When cycling through a turn, the rider’s center of mass travels a shorter path than the wheels because of the lean angle and, thus, moves at a reduced speed. To account for that difference during the calculation of $C_D A$ and $\mu$, we assumed that the track was circular with a circumference of 250 m, and that the wheels traveled a circular path with a radius that was greater than that of the center of mass by the height of the center of mass multiplied by the sine of the lean angle (where lean angle $= \arctan$ [centripetal acceleration/gravity]). We assumed that the height of the center of mass was equal to the height of the top of the saddle. Those assumptions would not provide accurate results at any specific point, but should provide a realistic approximation for the average data. We determined the normal force on the track surface as the vector sum of weight and centripetal force (using the circular-track assumption) and used the value associated with each speed and the acceleration.

During each trial, power and speed were recorded using field test procedure. Values did not differ. The relationships of modeled and measured speed for the maximal standing-start time trials were determined with linear regression. The 95% confidence interval for Pearson’s correlation coefficient was determined using the method of Fisher (6). A Bland–Altman plot was generated to allow inspection of the modeled and measured speed data.

RESULTS

Parameter determination and comparison. The $C_D A$ values determined from wind tunnel testing for each subject, interpolated to represent the yaw angles encountered during the cycling trials, averaged $0.261 \pm 0.006$ $m^2$, and individual values are shown in Table 1. Field-derived values of $C_D A$ averaged $0.258 \pm 0.006$ $m^2$ and did not differ ($P = 0.53$) from wind tunnel values (Table 1). The field-derived global coefficient of friction ($\mu$) while cycling on the taxiway was $0.0043 \pm 0.0006$.

Forward integration modeling of non–steady-state cycling performance. Field-derived values of $C_D A$ for the sprint cyclists were $0.245 \pm 0.044$ $m^2$ for the seated position and $0.304 \pm 0.055$ $m^2$ for the standing position, and individual values are reported in Table 2 and shown in Figure 1 as a sample data set of power versus speed. Subjects 2 and 3 used aerodynamic handlebars with elbow support for the seated position, whereas subject 1 used traditional racing handlebars. The mean value for $\mu$ was $0.0025 \pm 0.001$. Power, measured speed, and modeled speed for each subject during his or her time trial, which was a length appropriate to his or her specialization, are shown in Figure 2 (250 m for subject 1, a male match-sprint

<table>
<thead>
<tr>
<th>$C_D A$ ($m^2$)</th>
<th>Wind Tunnel</th>
<th>Field Derived</th>
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<tr>
<td>Subject 1</td>
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<td>Subject 6</td>
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specialist; 500 m for subject 3, a female 500-m time-trial specialist; and 1000 m for subject 2, a male kilometer time-trial specialist). Power produced by these world-class sprint cyclists was remarkably high, with a maximum value of 2517 W (for one revolution) for one of the male subjects. Predicted and measured speed for all subjects were highly correlated \((y = x, r^2 = 0.989, 95\% \text{ confidence interval for } r^2: 0.987-0.990, \text{ SEE} = 0.25)\). Additionally, a Bland–Altman plot (1) indicated agreement of the modeled and measured speed data (Fig. 3).

Performance changes that might result from changes in model parameters are presented in Table 3. Predicted time savings were small but possibly meaningful for internationally competitive sprint cyclists. Reduced mass produced greater time savings for the 250-m time trial, whereas reduced drag area produced greater time savings for the 500- and 1000-m time trials. In each scenario, reduced drag area and reduced mass additively improved performance. Decreased mass and power (a realistic scenario when reducing body mass) actually increased performance time (reduced performance).

DISCUSSION

Our two main findings were that field-derived values for modeling coefficients were equivalent to values derived from sophisticated wind tunnel testing, and that those coefficients could be used to accurately model speed during maximal-power, non–steady-state cycling with forward integration. These findings support our hypotheses and greatly broaden the application of our previous cycling model by eliminating the requirement for wind tunnel testing and by providing the means to assess maximal-power, non–steady-state cycling performance. Our new modeling technique can be used to quantify and possibly minimize drag area. Additionally, our model could be used to predict performance changes that might result from changes in various parameters (e.g., mass, equipment, frontal surface area).

Our finding that \(C_{DA}\) values determined in field trials were nearly identical to those determined in the wind tunnel is important because access to wind tunnel testing is both limited and expensive. Our data demonstrate that 5–10 steady-state cycling trials can be used to reproduce \(C_{DA}\) values obtained from wind tunnel testing. The mathematics we described are not particularly sophisticated.
Furthermore, we believe a technically skilled person could perform the field-based trials required to use our modeling procedures. Briefly, each bicycle must be equipped with an accurate power measurement device such as the SRM Powermeter or the Powertap (7). Wind speed and direction must be recorded during each trial, and elevation change across the test section (which influences change in potential energy) must be determined (surveys of most public road ways are generally available from local government offices). Finally, environmental conditions, including temperature, barometric pressure, and relative humidity must be recorded and used to determine air density. In the present study, we only evaluated the positions normally used by each subject; however, a sport scientist or cyclist could perform similar testing using a number of positions to identify a position that would minimize drag area. Thus, we believe that this work may have important practical applications because a cycling team could own a single power meter and optimize the aerodynamic drag of its members at relatively little expense. Such optimization could provide a substantial competitive edge in time-trial and triathlon events (11).

Our global coefficient of friction, \( \mu \), included rolling resistance and wheel-bearing friction, and therefore we expected it to be somewhat greater than previously reported values for coefficient of rolling resistance (\( C_{RR} \)). Our field-derived values for \( \mu \), 0.0042 for the taxayi and 0.0025 for the velodrome, confirmed our expectation. Our value of \( \mu \) for the track cyclists is larger than previous investigators have reported for \( C_{RR} \) of silk track tires on smooth surfaces of 0.0016 (9). However, the cyclists who participated in the second phase of this study used training tires for the steady-state trials, which have thicker casings and more durable tread compounds. Additionally, that coefficient is likely influenced by side loading or “tire scrub” associated with riding on the banked velodrome surface.

One particularly intriguing aspect of these data was the range of drag area values of the two male track sprint cyclists. Subject 1 was a match-sprint specialist with a body mass of 96 kg who used traditional racing handlebars. His drag area values (0.332 m\(^2\) seated and 0.414 m\(^2\) standing) were much greater than those for subject 2 (87 kg) who used aerodynamic handlebars and exhibited drag area values of 0.215 m\(^2\) when seated and 0.245 m\(^2\) when standing. Thus, the drag areas of subjects 1 and 2 differed by 68% when standing (i.e., effect of aerodynamic handlebars would be absent) even though their body mass differed by only 10%, and predicted body surface area (15) differed by only 4% (2.18 m\(^2\) vs 2.09 m\(^2\)). Consequently, the dramatic difference in drag area between these two world-class cyclists is not simply caused by body mass or surface area. Future research involving drag area and full anthropometric descriptions may provide a means for predicting body types with low drag area.

Our linear forward integration technique produced remarkably accurate predictions of speed during maximal standing-start acceleration and fatigue-related deceleration. The model produced similar predictive accuracy with cyclists who produced peak power outputs from 1377 to 2517 W and performed time trials from 250 to 1000 m in length. The accuracy was particularly satisfying because of the simple nature of the model and because of the capacities of our subjects. Specifically, our subjects were among the most successful track sprint cyclists in the world, and thus their acceleration likely represents the upper limits of human performance and a most severe modeling challenge. Even so, our modeling technique accounted for 99% of the variation in actual speed and predicted speed with a standard error of 0.25 ms\(^{-1}\). Such high precision in these extreme conditions suggests that our model will accurately predict cycling speed in most non-steady-state situations.

Modeling speed during velodrome cycling presented several unique challenges. First, the speed of the center of mass differed from the speed of the wheels when cycling in the turns because of lean angle. We modeled the velodrome with constant radius curves and straights. Such simple geometry almost certainly does not represent actual velodrome geometry; however, our attempts to obtain actual specifications from the track designers failed. Even so, our simple geometry was adequate to model speed with minimal errors. In addition, the center of aerodynamic pressure may not have been at the same position as the center of mass, and thus the appropriate air speed may have differed from the speed of the center of mass while in the turns. We did not attempt to account for this potential difference in our model. Indeed, we are unaware of any published values for the height of the center of pressure. Centripetal force increased normal force while cycling in the turns. To account for that increase, we modeled normal force (which determines rolling resistance) as the vector sum of gravity and centripetal force.
acting along the longitudinal axis of the rider. Velodrome surfaces are banked, even on the straight portions of the track. Because of this banking, the tires were subjected to side loading, which potentially increased the rolling resistance. We did not model the side loading in any specific way, but it is likely to contribute to our value for $\mu$ (as previously discussed). Finally, our velodrome model did not provide gradual transitions between the straights and the turns. We modeled the transitions as a 1-s (5 point) moving average in the sine of the lean angle. Although this transition did not exactly duplicate the speed changes at each turn entry and exit, it served as a reasonable approximation as shown in Figure 2.

One of the most important applications of our model may be in predicting the effects of changes in model parameters on cycling performance. With this in mind, we modeled four scenarios for each subject: 2% reduction in mass, 2% reduction in $C_{DA}$, 2% reductions in drag and power resulted in increased performance time for subjects 2 and 3 in the 500- and 1000-m time trials, respectively. Additionally, the predicted performance changes related to mass and drag were approximately additive, suggesting that mass and aerodynamic drag influence performance via different mechanisms such as initial acceleration (related to mass) and maximal speed (related to aerodynamic drag). Finally, the combination of reduced mass and power resulted in increased performance time for all three athletes, which underscores the potential dangers associated with reducing weight at the risk of decreasing power. Although these changes in performance time may seem small, they are large enough to have a substantial impact on final standings in world-class competition. For example, at the 2005 Track Cycling World Championships, the first three finishers in the women’s 500-m time trial were separated by 0.19 s, which is less than the difference our model predicts for a 2% decrease in aerodynamic drag. Similarly, the top two finishers in the men’s 1000-m time trial at that event were separated by only 0.065 s, which is less of an advantage than our model predicts for either the reduced mass or drag. Thus, the small but important improvements in performance predicted by our model should be given serious consideration by athletes, coaches, and sport scientists.

In summary, we have demonstrated that field-derived values for modeling coefficients were equivalent to those derived from sophisticated testing and that those coefficients can be used to accurately predict speed during maximal-power, non–steady-state cycling using linear forward integration. We believe that this modeling procedure will allow cyclists, coaches, and sport scientists to perform aerodynamic testing, optimization, and modeling without the use of a wind tunnel. Additionally, by using forward integration of known power–time profiles, cyclists can make realistic predictions of potential performance benefits or decrements from variations in any of the model parameters.

This study was only possible because of the cooperation of many people across three continents. We would like to thank the world champion athletes who donated their valuable training time, Prof. Allan Hahn and the staff in the physiology department at the AIS, Steve Hed who provided funding for the wind tunnel testing, and John Cobb for his technical assistance in the wind tunnel.

REFERENCES